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# New Equation for a Spin-1/2 Particle with Three Additional Characteristics in the Presence of Electromagnetic and Gravitational Fields

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**Abstract**
Within the general Gel'fand–Yaglom method, starting from the extended 28-component representation of the Lorentz group, we construct a new relativistic $P$-invariant generalized equation for a spin-1/2 particle possessing three characteristics in addition to the electric charge. The model is first developed for a free particle, where the corresponding system of spinor equations is derived and then transformed into spin-tensor form. In this form, we incorporate the interaction with external electromagnetic fields. By eliminating the accessory variables of the complete wave function, we obtain a minimal four-component Dirac-like equation that contains three new interaction terms, interpreted as arising from the additional electromagnetic characteristics of the particle. This approach is further extended to a Riemann space–time background within the conventional tetrad formalism, leading to additional geometrical interaction terms involving the Ricci scalar $R(x)$, the Ricci tensor $R_{\alpha\beta}$, and the Riemann curvature tensor $R_{\alpha\beta\rho\sigma}(x)$.

**Keywords:** spin-1/2 particle, relativistic symmetry, generalized equation, additional electromagnetic characteristics, external electromagnetic and gravitational fields

## 1. Introduction

The general theory of relativistic wave equations has a long history [1]–[26]; for more details, see the recent book [24]. Within the general method of Gel'fand–Yaglom [9], we consider an extended 28-component representation of the Lorentz group, comprising four bispinors and one third-rank spinor. This choice allows the construction of a relativistic system of equations for a spin-1/2 particle that possesses, in addition to electric charge, three further electromagnetic characteristics. The introduction of these characteristics extends the standard Dirac formalism by accommodating additional interaction structures, including higher-order derivatives, within a covariant framework.

We first work in Minkowski space and derive a four-component Dirac-like equation containing the additional interaction terms via couplings to the electromagnetic field tensor.

These terms are interpreted as corresponding to the three supplementary electromagnetic characteristics, and their structure goes beyond the minimal-coupling scheme by involving second-order derivatives. The resulting equation therefore generalizes the standard Dirac equation both in algebraic structure and in the types of physical interaction it describes.

The formalism is then extended to a Riemannian space–time background using the tetrad approach, where additional couplings to the curvature appear through the Ricci scalar, Ricci tensor, and Riemann tensor. Finally, the non-relativistic limit is developed for both flat and curved backgrounds, yielding generalized Pauli-type equations in which the same combination of electromagnetic parameters governs both magnetic and curvature-induced interactions. This framework thus provides a unified description of anomalous electromagnetic properties and geometric effects for spin-1/2 particles.

## 2. The new equation for a spin-1/2 particle

We construct a generalized relativistic equation for a spin-1/2 particle based on an extended 28-component set of irreducible representations of the proper Lorentz group.

$$T = 4\left[(0,1/2)\oplus(1/2,0)\right]\oplus\left[(1/2,1)\oplus(1,1/2)\right], \tag{2.1}$$

with the linking scheme

$$\begin{array}{ccc} 4(0,1/2) & - & 4(1/2,0) \\ | & & | \\ (1/2,1) & - & (1,1/2)\,. \end{array} \tag{2.2}$$

First, we construct a matrix equation for a free particle (applying the $ict$ -metric):

$$\left(\Gamma_\mu \partial_\mu + M\right)\Psi = 0, \quad \mu = 1,2,3,4\,. \tag{2.3}$$

In the modified Gel'fand–Yaglom basis, the matrix $\Gamma_4$ of the basic equation can be written in the form

$$\Gamma_4 = \begin{vmatrix} c^{(1/2)}\otimes\gamma_4 & 0 \\ 0 & c^{(3/2)}\otimes I_2 \otimes \gamma_4 \end{vmatrix}, \tag{2.4}$$

where the spin blocks $c^{(1/2)}$ and $c^{(3/2)}$ have the structure (corresponding to the linking scheme (2.2))

$$c^{(1/2)} = \begin{vmatrix} c_{11}^{(1/2)} & c_{12}^{(1/2)} & c_{13}^{(1/2)} & c_{14}^{(1/2)} & c_{15}^{(1/2)} \\ c_{21}^{(1/2)} & c_{22}^{(1/2)} & c_{23}^{(1/2)} & c_{24}^{(1/2)} & c_{25}^{(1/2)} \\ c_{31}^{(1/2)} & c_{32}^{(1/2)} & c_{33}^{(1/2)} & c_{34}^{(1/2)} & c_{35}^{(1/2)} \\ c_{41}^{(1/2)} & c_{42}^{(1/2)} & c_{43}^{(1/2)} & c_{44}^{(1/2)} & c_{45}^{(1/2)} \\ c_{51}^{(1/2)} & c_{52}^{(1/2)} & c_{53}^{(1/2)} & c_{54}^{(1/2)} & c_{55}^{(1/2)} \end{vmatrix}, \quad c^{(3/2)} = c_{55'}^{(3/2)}, \tag{2.5}$$

and $I_2$ is the $2\times 2$ unit matrix. The involved irreducible representations are enumerated as follows:

$$1,2,3,4 \Rightarrow (0,1/2), \quad 1',2',3',4' \Rightarrow (1/2,0), \quad 5 \Rightarrow (1,1/2), \quad 5' \Rightarrow (1/2,1). \tag{2.6}$$

Below we use shorter $\lambda$ and $\beta$ notations:

$$
\begin{gathered}
\lambda_1 = c_{11'}, \quad \lambda_2 = c_{22'}, \quad \lambda_3 = c_{33'}, \quad \lambda_4 = c_{44'}, \\
\frac{\beta_1}{\sqrt{2}} = -ic_{15'}, \quad \frac{\beta_2}{\sqrt{2}} = -ic_{25'}, \quad \frac{\beta_3}{\sqrt{2}} = -ic_{35'}, \quad \frac{\beta_4}{\sqrt{2}} = -ic_{45'}, \\
\frac{\beta_5}{\sqrt{2}} = ic_{51'}, \quad \frac{\beta_6}{\sqrt{2}} = ic_{52'}, \quad \frac{\beta_7}{\sqrt{2}} = ic_{53'}, \quad \frac{\beta_8}{\sqrt{2}} = ic_{54'}.
\end{gathered}
\tag{2.7}
$$

The involved parameters obey a number of quadratic constraints, which follow from standard physical requirements on the equation:

$$
\begin{gathered}
\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1, \\
\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 + \frac{3}{2}(\beta_1\beta_5 + \beta_2\beta_6 + \beta_3\beta_7 + \beta_4\beta_8) = 0, \\
\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4 + \\
\frac{3}{2}[(\lambda_2 + \lambda_3 + \lambda_4)\beta_1\beta_5 + (\lambda_1 + \lambda_3 + \lambda_4)\beta_2\beta_6 + (\lambda_1 + \lambda_2 + \lambda_4)\beta_3\beta_7 + (\lambda_1 + \lambda_2 + \lambda_3)\beta_4\beta_8] = 0, \\
\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4 + \\
\frac{3}{2}[(1-\lambda_1)\beta_1\beta_5 + (1-\lambda_2)\beta_2\beta_6 + (1-\lambda_3)\beta_3\beta_7 + (1-\lambda_4)\beta_4\beta_8] = 0, \\
2\lambda_1\lambda_2\lambda_3\lambda_4 + 3\beta_1\beta_5(\lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4) + 3\beta_2\beta_6(\lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_3\lambda_4) + \\
3\beta_3\beta_7(\lambda_1\lambda_2 + \lambda_1\lambda_4 + \lambda_2\lambda_4) + 3\beta_4\beta_8(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) = 0.
\end{gathered}
\tag{2.8}
$$

**3. The presence of electromagnetic fields**

We omit the technical details and start with the resulting first-order equations in the presence of external electromagnetic fields ($D_\mu = \partial_\mu - ieA_\mu$):

$$
\begin{gathered}
(M + \lambda_1\hat{D})\Psi^{(1)} - i2\beta_1\{D_\mu\Psi_\mu - \frac{1}{4}\hat{D}(\gamma_\mu\Psi_\mu)\} = 0, \\
(M + \lambda_2\hat{D})\Psi^{(2)} - i2\beta_2\{D_\mu\Psi_\mu - \frac{1}{4}\hat{D}(\gamma_\mu\Psi_\mu)\} = 0, \\
(M + \lambda_3\hat{D})\Psi^{(3)} - i2\beta_3\{D_\mu\Psi_\mu - \frac{1}{4}\hat{D}(\gamma_\mu\Psi_\mu)\} = 0, \\
(M + \lambda_4\hat{D})\Psi^{(4)} - i2\beta_4\{D_\mu\Psi_\mu - \frac{1}{4}\hat{D}(\gamma_\mu\Psi_\mu)\} = 0, \\
-i\left(D_\lambda - \frac{1}{4}\gamma_\lambda\hat{D}\right)(\beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}) + M\{\Psi_\lambda - \frac{1}{4}\gamma_\lambda(\gamma_\mu\Psi_\mu)\} = 0
\end{gathered}
\tag{3.1}
$$

where the four bispinors and one vector–bispinor are defined as

$$
\Psi^{(k)} = \begin{vmatrix} \Psi^{(k)\dot{a}} \\ \Psi^{(k)}_a \end{vmatrix}, \quad \Psi_\mu = \begin{vmatrix} \Psi^{\dot{a}}_\mu \\ \Psi_{a\mu} \end{vmatrix}, \quad k = 1,2,3,4. \tag{3.2}
$$

The linear combination of four bispinors is denoted by $\Psi$:

$$
\Psi = \beta_5\Psi^{(1)} + \beta_6\Psi^{(2)} + \beta_7\Psi^{(3)} + \beta_8\Psi^{(4)}. \tag{3.3}
$$

From these equations we can derive (with $\hat{D} = \gamma_\mu D_\mu$):

$$\left(M+\lambda_1 D\right)\Psi^1+\frac{2\beta_1}{M}\left(D^2-\frac{1}{4}\hat{D}\hat{D}\right)\Psi=0, \tag{3.4}$$

$$\left(M+\lambda_2 D\right)\Psi^2+\frac{2\beta_2}{M}\left(D^2-\frac{1}{4}\hat{D}\hat{D}\right)\Psi=0, \tag{3.5}$$

$$\left(M+\lambda_3 D\right)\Psi^3+\frac{2\beta_3}{M}\left(D^2-\frac{1}{4}\hat{D}\hat{D}\right)\Psi=0, \tag{3.6}$$

$$\left(M+\lambda_4 D\right)\Psi^4+\frac{2\beta_4}{M}\left(D^2-\frac{1}{4}\hat{D}\hat{D}\right)\Psi=0. \tag{3.7}$$

Next, we act

$$\text{on 3.4 by } \beta_5\left(M+\lambda_2\hat{D}\right)\left(M+\lambda_3\hat{D}\right)\left(M+\lambda_4\hat{D}\right), \tag{3.8}$$

$$\text{on 3.5 by } \beta_6\left(M+\lambda_1\hat{D}\right)\left(M+\lambda_3\hat{D}\right)\left(M+\lambda_4\hat{D}\right), \tag{3.9}$$

$$\text{on 3.6 by } \beta_7\left(M+\lambda_1\hat{D}\right)\left(M+\lambda_2\hat{D}\right)\left(M+\lambda_4\hat{D}\right), \tag{3.10}$$

$$\text{on 3.7 by } \beta_8\left(M+\lambda_1\hat{D}\right)\left(M+\lambda_2\hat{D}\right)\left(M+\lambda_3\hat{D}\right), \tag{3.11}$$

and sum the results. Using the identities

$$\hat{D}\hat{D}=D^2-ieF_{\rho\lambda}J_{\rho\lambda},\quad D^2=D_\mu D_\mu,\quad \gamma_\mu\gamma_\nu\gamma_\rho=\delta_{\mu\nu}\gamma_\rho-\delta_{\mu\rho}\gamma_\nu+\delta_{\nu\rho}\gamma_\mu+\epsilon_{\mu\nu\rho\eta}\gamma_5\gamma_\eta, \tag{3.12}$$

and taking into account the constraints on the $\lambda$ parameters, we obtain the basic equation:

$$\Bigg\{M+\gamma_\rho D_\rho-\frac{ie}{M}\mu F_{\mu\beta}J_{\mu\beta}-\frac{ie}{M^2}\sigma\hat{D}F_{\mu\beta}J_{\mu\beta}+$$

$$\frac{1}{M^3}\eta\left(-ie^2D^2F_{\alpha\beta}J_{\alpha\beta}-e^2F_{\alpha\beta}F_{\beta\rho}\gamma_\alpha\gamma_\rho-e^2\frac{1}{2}F_{\alpha\beta}F_{\alpha\beta}-e^2\frac{1}{4}\gamma_5\epsilon_{\rho\lambda\alpha\beta}F_{\rho\lambda}F_{\alpha\beta}\right)\Bigg\}\Psi=0, \tag{3.13}$$

where

$$\mu=\frac{4}{3}\left(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4\right), \tag{3.14}$$

$$\sigma=\frac{4}{3}\left(\lambda_1\lambda_2\lambda_3+\lambda_1\lambda_2\lambda_4+\lambda_1\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_4\right),\quad \eta=\frac{4}{3}\lambda_1\lambda_2\lambda_3\lambda_4. \tag{3.15}$$

This equation describes a spin-$1/2$ particle which, in addition to its electric charge, possesses three additional electromagnetic characteristics $\mu$, $\sigma$, and $\eta$. The structure of the resulting equation differs significantly from the known Dirac equation because it contains second-order derivatives in the additional interaction terms.

**4. The presence of gravitational fields**

Assuming the use of the relativistic interval in real-valued form $ds^2=c^2dt^2-dx^2-dy^2-dz^2$, one should use the following form of the basic equation in flat Minkowski space:

$$\Bigg\{i\gamma^\rho D_\rho-M+\frac{e\mu}{M}F_{\alpha\beta}j^{\alpha\beta}+\frac{e\sigma}{M^2}\hat{D}F_{\alpha\beta}j^{\alpha\beta}+$$

$$\frac{\eta}{M^3}\left(-ie^2D^2F_{\alpha\beta}j^{\alpha\beta}-e^2\gamma^\alpha F_{\alpha\beta}F^\beta_\rho\gamma^\rho-e^2\frac{1}{2}F_{\alpha\beta}F^{\alpha\beta}-e^2\frac{1}{4}\gamma_5\epsilon^{\rho\lambda\alpha\beta}F_{\rho\lambda}F_{\alpha\beta}\right)\Bigg\}\Psi=0. \tag{4.1}$$

We now extend this approach to a space–time models with Riemannian structure. To this end, we start with the system:

$$\left(M+\lambda_1\hat{D}\right)\Psi^{(1)}-2i\beta_1\left(D^{\mu}\Psi_{\mu}-\frac{1}{4}\hat{D}\left(\gamma^{\mu}\Psi_{\mu}\right)\right)=0, \tag{4.2}$$

$$\left(M+\lambda_2\hat{D}\right)\Psi^{(2)}-2i\beta_2\left(D^{\mu}\Psi_{\mu}-\frac{1}{4}\hat{D}\left(\gamma^{\mu}\Psi_{\mu}\right)\right)=0, \tag{4.3}$$

$$\left(M+\lambda_3\hat{D}\right)\Psi^{(3)}-2i\beta_3\left(D^{\mu}\Psi_{\mu}-\frac{1}{4}\hat{D}\left(\gamma^{\mu}\Psi_{\mu}\right)\right)=0, \tag{4.4}$$

$$\left(M+\lambda_4\hat{D}\right)\Psi^{(4)}-2i\beta_4\left(D^{\mu}\Psi_{\mu}-\frac{1}{4}\hat{D}\left(\gamma^{\mu}\Psi_{\mu}\right)\right)=0, \tag{4.5}$$

$$-i\left(D_{\lambda}-\frac{1}{4}\gamma_{\lambda}\hat{D}\right)\left(\beta_5\Psi^{(1)}+\beta_6\Psi^{(2)}+\beta_7\Psi^{(3)}+\beta_8\Psi^{(4)}\right)+M\left(\Psi_{\lambda}-\frac{1}{4}\gamma_{\lambda}\left(\gamma^{\mu}\Psi_{\mu}\right)\right)=0, \tag{4.6}$$

where $\Psi^{(a)}$, $a=1,2,3,4$ are covariant bispinors, and $\Psi_{\mu}$ is a covariant vector–bispinor. We apply the generalized derivative $D_{\mu}$, which accounts for the presence of both electromagnetic fields and a curved space–time background. The symbol $\nabla_{\mu}$ denotes the covariant derivative, $\Gamma_{\mu}(x)$ is the bispinor connection, and $\gamma_{\mu}(x)$ are the local Dirac matrices:

$$D_{\mu}=\nabla_{\mu}-ieA_{\mu}(x)+\Gamma_{\mu}(x),\quad \hat{D}=\gamma^{\mu}D_{\mu}=\gamma^{\mu}(x)\left(\nabla_{\mu}-ieA_{\mu}+\Gamma_{\mu}\right). \tag{4.7}$$

We have

$$\begin{gathered}\hat{D}\hat{D}=\left(\gamma^{\alpha}D_{\alpha}\right)\left(\gamma^{\beta}D_{\beta}\right)=D_{\alpha}\frac{\gamma^{\alpha}\gamma^{\beta}+\gamma^{\beta}\gamma^{\alpha}}{2}D_{\beta}+D_{\alpha}\frac{\gamma^{\alpha}\gamma^{\beta}-\gamma^{\beta}\gamma^{\alpha}}{2}D_{\beta}= \\ g^{\alpha\beta}(x)D_{\alpha}D_{\beta}=D^{\alpha}D_{\alpha}+j^{\alpha\beta}(x)[D_{\alpha},D_{\beta}]_{-}=D^2+\sigma^{\alpha\beta}(x)M_{\alpha\beta}(x),\end{gathered} \tag{4.8}$$

where

$$D^2=D^{\alpha}D_{\alpha},M_{\alpha\beta}(x)=[D_{\alpha},D_{\beta}]_{-},\quad j^{\alpha\beta}(x)=\frac{\gamma^{\alpha}(x)\gamma^{\beta}(x)-\gamma^{\beta}(x)\gamma^{\alpha}(x)}{4}. \tag{4.9}$$

We then derive

$$\left\{M+\hat{D}+\frac{\mu}{M}j^{\alpha\beta}M_{\alpha\beta}+\frac{\sigma}{M^2}\hat{D}j^{\alpha\beta}M_{\alpha\beta}+\frac{\eta}{M^3}\left(D^2j^{\alpha\beta}M_{\alpha\beta}+j^{\rho\sigma}j^{\alpha\beta}M_{\rho\sigma}M_{\alpha\beta}\right)\right\}\Psi=0, \tag{4.10}$$

where the involved additional terms are

$$M_{\alpha\beta}\Psi=\left(D_{\alpha}D_{\beta}-D_{\beta}D_{\alpha}\right)\Psi=\left(ieF_{\alpha\beta}+\frac{1}{2}j^{\nu\rho}R_{\nu\rho\alpha\beta}\right)\Psi, \tag{4.11}$$

$$\frac{\mu}{M}j^{\alpha\beta}M_{\alpha\beta}\Psi=\frac{\mu}{M}\left(ieF_{\alpha\beta}j^{\alpha\beta}-\frac{1}{4}R(x)\right)\Psi, \tag{4.12}$$

$$\frac{\sigma}{M^2}\hat{D}j^{\alpha\beta}M_{\alpha\beta}\Psi=\frac{1}{M^2}\sigma\left(\gamma^{\rho}D_{\rho}\right)\left(ieF_{\alpha\beta}j^{\alpha\beta}-\frac{1}{4}R(x)\right)\Psi, \tag{4.13}$$

$$\begin{gathered}\frac{\eta}{M^3}j^{\rho\sigma}j^{\alpha\beta}M_{\rho\sigma}M_{\alpha\beta}=\frac{\eta}{M^3}\Big[(ieF_{\rho\sigma}j^{\rho\sigma}-\frac{1}{4}R)^2+ \\ \frac{ie}{2}F_{\alpha\beta}\left(-2j^{\rho\alpha}R^{\beta}_{\rho}-2j^{\nu\rho}R^{\beta\alpha}_{\nu\rho}-2i\gamma^5j^{\rho\delta}\grave{o}^{\sigma\alpha\nu}_{\delta}R^{\beta}_{\nu\rho\sigma}\right)+R^{\beta}_{\nu\rho\sigma}R_{\grave{o}\tau\alpha\beta}j^{\rho\sigma}j^{\alpha\nu}j^{\grave{o}\tau}\Big].\end{gathered} \tag{4.14}$$

Thus, the final form of the basic equation is:

$$\Big\{\left(\gamma^{\sigma}D_{\sigma}+M\right)+\frac{\mu}{M}\left(ieF_{\alpha\beta}j^{\alpha\beta}-\frac{1}{4}R\right)+\frac{\sigma}{M^{2}}\left(\gamma^{\rho}D_{\rho}\right)\left(ieF_{\alpha\beta}j^{\alpha\beta}-\frac{1}{4}R\right)+$$

$$\frac{\eta}{M^{3}}\Big[D^{\sigma}D_{\sigma}\left(ieF_{\alpha\beta}j^{\alpha\beta}-\frac{1}{4}R\right)+\left(ieF_{\rho\sigma}j^{\rho\sigma}-\frac{1}{4}R\right)^{2}+$$

$$\frac{ie}{2}F_{\alpha\beta}\left(2j^{\alpha\rho}R^{\beta}_{\rho}-2j^{\nu\rho}R^{\beta\,\alpha}_{\nu\rho}-2i\gamma^{5}j^{\rho\delta}\grave{o}^{\alpha\nu\sigma}_{\delta}R^{\beta}_{\nu\sigma\rho}\right)-\left(R^{\beta}_{\nu\rho\sigma}j^{\rho\sigma}\right)j^{\nu\alpha}\left(R_{\alpha\beta\grave{o}\tau}j^{\grave{o}\tau}\right)\Big]\Big\}\Psi=0. \quad (4.15)$$

In order to have a standard presentation for Dirac equation, we should multiply the above equation by imaginary unit $i$, and make replacement $M \Rightarrow iM$, so, we obtain

$$\Big\{\left(i\gamma^{\sigma}D_{\sigma}-M\right)+\frac{\mu}{M}\left(ieF_{\alpha\beta}j^{\alpha\beta}-\frac{1}{4}R\right)+\frac{-i\sigma}{M^{2}}\left(\gamma^{\rho}D_{\rho}\right)\left(ieF_{\alpha\beta}j^{\alpha\beta}-\frac{1}{4}R\right)+$$

$$\frac{-\eta}{M^{3}}\Big[D^{\sigma}D_{\sigma}\left(ieF_{\alpha\beta}j^{\alpha\beta}-\frac{1}{4}R\right)+\left(ieF_{\rho\sigma}j^{\rho\sigma}-\frac{1}{4}R\right)^{2}+$$

$$\frac{ie}{2}F_{\alpha\beta}\left(2j^{\alpha\rho}R^{\beta}_{\rho}-2j^{\nu\rho}R^{\beta\,\alpha}_{\nu\rho}-2i\gamma^{5}j^{\rho\delta}\grave{o}^{\alpha\nu\sigma}_{\delta}R^{\beta}_{\nu\sigma\rho}\right)-\left(R^{\beta}_{\nu\rho\sigma}j^{\rho\sigma}\right)j^{\nu\alpha}\left(R_{\alpha\beta\grave{o}\tau}j^{\grave{o}\tau}\right)\Big]\Big\}\Psi=0. \quad (4.16)$$

Equation (4.16) differs from (4.15) only in the formal change of notations:

$$\mu=\mu, \quad \sigma\Rightarrow -i\sigma, \quad \eta\Rightarrow -\eta, \quad M>0, \qquad (4.17)$$

till now the symbols $\mu,\sigma,\eta$ are just formal notations; some of them may be even imaginary (see in the end of the next Section)

As seen, a number of additional geometrical interaction terms arise through the Ricci scalar $R(x)$, the Ricci tensor $R_{\alpha\beta}$, and the Riemann curvature tensor $R_{\alpha\beta\rho\sigma}(x)$. The contributions from the Ricci tensor and Riemann tensor are zero only if the third parameter $\eta$ vanishes.

## 5. The non-relativistic equation in flat space

Let us perform the $(3+1)$-splitting of the above equation. Using the identities

$$F_{l}^{n}F_{n}^{l}=2\left(\vec{E}^{2}-\vec{B}^{2}\right), \quad F_{kl}F_{n}^{l}j^{kn}\equiv 0,$$
$$\frac{1}{2}F_{kl}F^{kl}=\vec{B}^{2}-\vec{E}^{2}, \quad \frac{1}{4}\gamma_{5}\,\varepsilon^{mnkl}F_{mn}F_{kl}=\gamma^{5}\vec{E}\,\vec{B}, \qquad (5.1)$$

and the notations $(K_i)=\left(j^{01},j^{02},j^{03}\right)$, $(J_i)=\left(j^{23},j^{31},j^{12}\right)$, we can write the main equation in the form

$$\{i\left(\gamma^{0}D_{0}+\gamma^{j}D_{j}\right)-M+\frac{2ei\mu}{M}\left(\vec{E}\,\vec{K}+\vec{B}\vec{J}\right)+\frac{2e\sigma}{M^{2}}\left(\gamma^{0}D_{0}+\gamma^{j}D_{j}\right)\left(\vec{E}\,\vec{K}+\vec{B}\vec{J}\right)-$$
$$\frac{e\eta}{M^{3}}\left(2i\left(D_{0}^{2}-D_{j}D_{j}\right)\left(\vec{E}\,\vec{K}+\vec{B}\vec{J}\right)+\left(\vec{E}^{2}-\vec{B}^{2}\right)+\gamma^{5}\vec{E}\,\vec{B}\right)\Psi=0. \qquad (5.2)$$

It is convenient to use the Pauli representation for the Dirac matrices:

$$\gamma^{0}=\begin{vmatrix} I & 0 \\ 0 & -I \end{vmatrix}, \quad \gamma^{j}=\begin{vmatrix} 0 & \sigma_{j} \\ -\sigma_{j} & 0 \end{vmatrix}, \quad \gamma^{5}=-i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}=\begin{vmatrix} 0 & -I \\ -I & 0 \end{vmatrix}, \qquad (5.3)$$

$$\sigma_{0}=\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \quad \sigma_{1}=\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \quad \sigma_{2}=\begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}, \quad \sigma_{3}=\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}. \qquad (5.4)$$

The generators $j^{ab} = \frac{1}{4}\left(\gamma^a\gamma^b - \gamma^b\gamma^a\right)$ are given by

$$K_1 = \frac{1}{2}\begin{vmatrix}\sigma_1 & 0\\ 0 & -\sigma_1\end{vmatrix},\quad K_2 = \frac{1}{2}\begin{vmatrix}\sigma_2 & 0\\ 0 & -\sigma_2\end{vmatrix},\quad K_3 = \frac{1}{2}\begin{vmatrix}\sigma_3 & 0\\ 0 & -\sigma_3\end{vmatrix}, \tag{5.5}$$

$$J_1 = -\frac{i}{2}\begin{vmatrix}\sigma_1 & 0\\ 0 & \sigma_1\end{vmatrix},\quad J_2 = -\frac{i}{2}\begin{vmatrix}\sigma_2 & 0\\ 0 & \sigma_2\end{vmatrix},\quad J_3 = -\frac{i}{2}\begin{vmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{vmatrix}. \tag{5.6}$$

To define large and small components, we apply the projection operators:

$$P_+ = P_1 = \frac{I+\gamma^0}{2} = \begin{vmatrix}I & 0\\ 0 & 0\end{vmatrix},\quad P_- = P_2 = \frac{I-\gamma^0}{2} = \begin{vmatrix}0 & 0\\ 0 & I\end{vmatrix}, \tag{5.7}$$

we write

$$\Psi = \begin{vmatrix}\varphi_+(x)\\ \varphi_-(x)\end{vmatrix},\quad \Psi_1 = P_1\Psi = \begin{vmatrix}\varphi_+(x)\\ 0\end{vmatrix},\quad \Psi_2 = P_2\Psi = \begin{vmatrix}0\\ \varphi_-(x)\end{vmatrix}. \tag{5.8}$$

In the non-relativistic limit, $\varphi_+(x)$ is the large component and $\varphi_-(x)$ the small one.

Presenting the main equation in block form, we obtain the coupled equations:

$$\begin{gathered}
\left(iD_0 - M\right)\varphi_+ + iD_k\sigma_k\varphi_- + \frac{ei\mu}{M}\left(\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_+ +\\
\frac{e\sigma}{M^2}\left[D_0\left(\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_+ + D_k\sigma_k\left(-\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_-\right] -\\
-\frac{e\eta}{M^3}\left[\left(D_0^2 - D_jD_j\right)\left(\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_+ + \left(\vec{E}^2 - \vec{B}^2\right)\varphi_+ - \left(\vec{E}\,\vec{B}\right)\varphi_-\right] = 0,
\end{gathered} \tag{5.9}$$

$$\begin{gathered}
-iD_k\sigma_k\varphi_+ + \left(-iD_0 - M\right)\varphi_- + \frac{ei\mu}{M}\left(-\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_- +\\
\frac{e\sigma}{M^2}\left[-D_k\sigma_k\left(\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_+ - D_0\left(-\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_-\right] -\\
\frac{e\eta}{M^3}\left[\left(D_0^2 - D_jD_j\right)\left(-\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_- + \left(\vec{E}^2 - \vec{B}^2\right)\varphi_- - \left(\vec{E}\,\vec{B}\right)\varphi_+\right] = 0.
\end{gathered} \tag{5.10}$$

We now separate the rest energy by the substitutions:

$$D_0 \Rightarrow \left(D_0 - iM\right),\quad iD_0 \Rightarrow \left(iD_0 + M\right),\quad D_0^2 \Rightarrow \left(D_0^2 - 2iMD_0 - M^2\right). \tag{5.11}$$

We then obtain

$$\begin{gathered}
i\frac{1}{M}D_0\varphi_+ + i\frac{1}{M}D_k\sigma_k\varphi_- + \frac{ei\mu}{M^2}\left(\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_+ +\\
\frac{e\sigma}{M^2}\left[\left(\frac{1}{M}D_0 - i\right)\left(\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_+ + \frac{1}{M}D_k\sigma_k\left(-\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_-\right] -\\
\frac{e\eta}{M^2}\left[\left(\frac{1}{M^2}D_0^2 - 2i\frac{1}{M}D_0 - 1 - \frac{1}{M^2}D_jD_j\right)\left(\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_+ +\right.\\
\left. + \frac{1}{M^2}\left(\vec{E}^2 - \vec{B}^2\right)\varphi_+ - \frac{1}{M^2}\left(\vec{E}\,\vec{B}\right)\varphi_-\right] = 0,
\end{gathered} \tag{5.12}$$

$$-i\frac{1}{M}D_k\sigma_k\varphi_+ + \left(-i\frac{1}{M}D_0 - 2\right)\varphi_- + \frac{ei\mu}{M^2}\left(-\vec{E}\vec{\sigma} - i\vec{B}\vec{\sigma}\right)\varphi_- +$$

$$\frac{e\sigma}{M^2}\left[-\frac{1}{M}D_k\sigma_k\left(\vec{E}\vec{\sigma}-i\vec{B}\vec{\sigma}\right)\varphi_+ -\left(\frac{1}{M}D_0-i\right)\left(-\vec{E}\vec{\sigma}-i\vec{B}\vec{\sigma}\right)\varphi_-\right]-$$

$$\frac{e\eta}{M^2}\left[\left(\frac{1}{M^2}D_0^2-2i\frac{1}{M}D_0-1-\frac{1}{M^2}D_jD_j\right)\left(-\vec{E}\vec{\sigma}-i\vec{B}\vec{\sigma}\right)\varphi_- +\right.$$

$$\left.\left(\vec{E}^2-\vec{B}^2\right)\varphi_- -\frac{1}{M^2}\left(\vec{E}\,\vec{B}\right)\varphi_+\right]=0. \qquad (5.13)$$

It is known (for instance, see [3, 4]) that we should assume the following orders of smallness for the involved quantities (magnetic components $B_j$ arise from commutators $D_{[kl]}$, electric components $E_k$ from commutators $D_{[0k]}$, whence follow their smallness orders):

$$\varphi_+\sim 1, \varphi_-\sim x, \quad \frac{1}{M}D_j\sim x, \quad \frac{1}{M}D_0\sim x^2, \quad \frac{B_j}{M^2}\sim x^2, \quad \frac{E_j}{M^2}\sim x^3. \qquad (5.14)$$

In both equations, we will preserve only the terms of order $x$ and $x^2$. In this way we obtain:

$$x^2: \quad i\frac{1}{M}D_0\varphi_+ + i\frac{1}{M}D_k\sigma_k\varphi_- +\frac{e\mu}{M^2}\left(\vec{B}\vec{\sigma}\right)\varphi_+ -\frac{e\sigma}{M^2}\left(\vec{B}\vec{\sigma}\right)\varphi_+ -i\frac{e\eta}{M^2}\left(\vec{B}\vec{\sigma}\right)\varphi_+ =0; \qquad (5.15)$$

$$x: \quad \varphi_- = -i\frac{1}{2M}D_k\sigma_k\varphi_+ =0. \qquad (5.16)$$

Eliminating the small component $\varphi_-$, we derive (changing the notations $\sigma\Rightarrow i\sigma,\ \eta\Rightarrow-\eta$):

$$iD_0\varphi_+ +\frac{1}{2M}\left(D_k\sigma_k\right)\left(D_n\sigma_n\right)\varphi_+ +\frac{e}{M}\left(+\mu-\sigma-i\eta\right)\left(\vec{B}\vec{\sigma}\right)\varphi_+ =0. \qquad (5.17)$$

Keeping in mind the multiplication rule for Pauli matrices, we arrive at the equation (let $\psi=\Psi_+$, by physical reason we should assume the replacements: $\mu\Rightarrow\mu,\ \sigma=\sigma,\ -i\eta\Rightarrow\eta$):

$$iD_0 red\psi+\frac{1}{2M}D^2\psi+\frac{e}{2M}\left(\vec{B}\vec{\sigma}\right)\psi+\frac{e}{M}\left(\mu+\sigma+\eta\right)\vec{B}\vec{\sigma}\psi=0. \qquad (5.18)$$

Thus, in the non-relativistic limit, the generalized equation takes the form of the ordinary Pauli equation for a spin-1/2 particle with anomalous magnetic moment due to additional $(\mu+\sigma+\eta)$-term.

**6. The non-relativistic approximation in presence of curved space background**

The detailed structure of the basic equation within the tetrad formalism reads as

$$\left\{i\gamma^c(e^{\sigma}_{(c)}\left(\partial_\sigma+ieA_\sigma+\Gamma_\sigma\right)-M+\frac{\mu}{M}\left(ieF_{[kl]}j^{[kl]}-\frac{1}{4}R\right)+\right.$$

$$\frac{-i\sigma}{M^2}\gamma^c\left(e^{\sigma}_{(c)}\partial_\sigma+\Gamma_c\right)\left(ieF_{[kl]}j^{[kl]}-\frac{1}{4}R\right)+$$

$$\frac{-\eta}{M^3}\left[D^\sigma D_\sigma\left(ieF_{[kl]}j^{[kl]}-\frac{1}{4}R\right)+(ieF_{[kl]}j^{[kl]}-\frac{1}{4}R)^2+\right.$$

$$\frac{ie}{2}F_{[kl]}\left(\left(j^{[kc]}R^{\ l}_{c}-j^{[lc]}R^{\ k}_{c}\right)-j^{[nc]}\left(R^{lk}_{nc}-R^{lk}_{cn}\right)-i\gamma^5 j^{[cd]}\grave{o}_d^{\ kns}\left(R^l_{\ nsc}-R^l_{\ snc}\right)\right)-$$

$$\left.\left.\left(R^b_{\ ncs}j^{[cs]}\right)j^{[na]}\left(R_{abkl}\,j^{[kl]}\right)\right]\right\}\Psi=0, \qquad (6.1)$$

where $D_c = e^{\sigma}_{(c)}\partial_\sigma + ieA_c + \frac{1}{2}\gamma_{[mn]c} j^{[mn]}, e^{\sigma}_{(c)}$ is a tetred and the symbols $\gamma_{[mn]c}$ stand for Ricci rotation coefficients. The Latin letters designate the tetrad components. Equation (6.1) contains the scalar and tensor Ricci quantities.

In order to develop the non-relativistic approximation (this is possible only for the non-relativistic metric, $dS^2 = dx_0^2 + g_{kl}dx^l dx^l$, we need to fix the smallness orders of the involved geometrical quantities:

$$R_{abcd} = \left(\gamma_{abc,d} - \gamma_{abd,c}\right) + \left(\gamma_{abf}\gamma^f_{cd} - \gamma_{abf}\gamma^f_{dc}\right) + \left(\gamma_{afc}\gamma^f_{bd} - \gamma_{afd}\gamma^f_{bc}\right), \tag{6.2}$$

$$R_{mn} = R^a_{mcn} = \left(\gamma^c_{mc,n} - \gamma^c_{mn,c}\right) + \left(\gamma^c_{bf}\gamma^f_{cd} - \gamma^c_{bf}\gamma^f_{dc}\right) + \left(\gamma^c_{fc}\gamma^f_{bd} - \gamma^c_{fd}\gamma^f_{bc}\right). \tag{6.3}$$

In the non-relativistic equations, only the components of the Ricci tensor with spatial indices are present, so we get the simpler formula

$$R_{bd} = \left(\gamma^i_{bi,d} - \gamma^i_{bd,i}\right) + \left(\gamma^i_{bf}\gamma^f_{id} - \gamma^i_{bf}\gamma^f_{di}\right) + \left(\gamma^i_{fi}\gamma^f_{bd} - \gamma^i_{fd}\gamma^f_{bi}\right). \tag{6.4}$$

In the Ricci rotation coefficients, only the following appear: $\gamma_{[ij]0}$ and $\gamma_{[ij]k}$. Therefore, expressions for the generalized derivatives are simplified (note that $n, m, k, l \in \{1,2,3\}$):

$$D_0 = \partial_0 + ieA_0 + \frac{1}{2}\gamma_{[kl]0} j^{[kl]}, \quad D_m = e^n_{(m)}\left(\partial_n + ieA_n\right) + \frac{1}{2}\gamma_{[kl]m} j^{[kl]}. \tag{6.5}$$

So, in the non-relativistic metric, we have:

$$R_{00} = 0, \quad R_{01} = 0, \quad R_{02} = 0, \quad R_{02} = 0, \tag{6.6}$$

$$\begin{aligned}
R_{11} &= (\gamma^i_{1i,1} - \gamma^i_{11,i}) + (\gamma^i_{1k}\gamma^k_{i1} - \gamma^i_{1k}\gamma^k_{1i}) + (\gamma^i_{ki}\gamma^k_{11} - \gamma^i_{k1}\gamma^k_{1i}),\\
R_{22} &= (\gamma^i_{2i,2} - \gamma^i_{22,i}) + (\gamma^i_{2k}\gamma^k_{i2} - \gamma^i_{2k}\gamma^k_{2i}) + (\gamma^i_{ki}\gamma^k_{22} - \gamma^i_{k2}\gamma^k_{2i}),\\
R_{33} &= (\gamma^i_{3i,3} - \gamma^i_{33,i}) + (\gamma^i_{3k}\gamma^k_{i3} - \gamma^i_{3k}\gamma^k_{3i}) + (\gamma^i_{ki}\gamma^k_{33} - \gamma^i_{k3}\gamma^k_{3i}),\\
R_{23} &= (\gamma^i_{2i,3} - \gamma^i_{23,i}) + (\gamma^i_{2k}\gamma^k_{i3} - \gamma^i_{2k}\gamma^k_{3i}) + (\gamma^i_{ki}\gamma^k_{23} - \gamma^i_{k3}\gamma^k_{2i}),\\
R_{31} &= (\gamma^i_{3i,1} - \gamma^i_{31,i}) + (\gamma^i_{3k}\gamma^k_{i1} - \gamma^i_{3k}\gamma^k_{1i}) + (\gamma^i_{ki}\gamma^k_{31} - \gamma^i_{k1}\gamma^k_{3i}),\\
R_{12} &= (\gamma^i_{1i,2} - \gamma^i_{12,i}) + (\gamma^i_{1k}\gamma^k_{i2} - \gamma^i_{1k}\gamma^k_{2i}) + (\gamma^i_{ki}\gamma^k_{12} - \gamma^i_{k2}\gamma^k_{1i}).
\end{aligned} \tag{6.7}$$

Similarly, for non-vanishing components of the curvature tensor we have (indices belong to $\{1,2,3\}$):

$$R_{klmn} = \left(\gamma_{klm,n} - \gamma_{kln,m}\right) + \left(\gamma_{klj}\gamma^j_{mn} - \gamma_{klj}\gamma^j_{nm}\right) + \left(\gamma_{kjm}\gamma^j_{ln} - \gamma_{kjn}\gamma^j_{lm}\right). \tag{6.8}$$

The smallness orders of the involved quantities are:

$$\begin{aligned}
&\frac{D_n}{M} \sim \frac{\gamma_{kln}}{M} \sim x, \quad \frac{D_0}{M} \sim x^2, \quad \frac{B_n}{M^2} \sim x^2, \quad \frac{B_n^2}{M^4} \sim x^4, \quad \frac{E_n}{M^2} \sim x^3, \quad \frac{E_n^2}{M^4} \sim x^6,\\
&\frac{\gamma_{kl0}}{M} \sim x^2, \quad \frac{R_{kl}}{M^2} \sim x^2, \quad \frac{R}{M^2} \sim x^2, \quad \frac{R_{klmn}}{M^2} \sim x^2, \quad \frac{R_{klmn}}{M^2} \times \frac{R_{klmn}}{M^2} \sim x^4.
\end{aligned} \tag{6.9}$$

Further, making the needed calculations, we arrive at a generalized Pauli-like equation

$$\begin{aligned}
\left(i\partial_0 - eA_0(x) + \frac{1}{2}G_{n0}(x)\sigma_n\right)\Psi = -\frac{1}{2M}(\sigma_m e^n_{(m)}(x)\left(\partial_n + ieA_n(x)\right) -\\
\frac{i}{2}\sigma_m\sigma_n G_{mn}(x))^2\Psi + \frac{1}{M}(\mu + \sigma + \eta)\left(eB_n\sigma_n - \frac{1}{4}R\right)\Psi .
\end{aligned} \tag{6.10}$$

where we apply shortening notations for Ricci rotation coefficients:

$$\left(\gamma_{[01]0},\gamma_{[02]0},\gamma_{[03]0}\right)=G_{j0}\left(x\right),\left(\gamma_{[23]m},\gamma_{[02]m},\gamma_{[03]m}\right)=G_{nm}\left(x\right).$$

Thus, in the presence of a curved space–time background, the Pauli-like equation takes the form of the ordinary Pauli equation for a spin-1/2 particle with anomalous magnetic moment $\left(\mu+\sigma+\eta\right)$, and the same coefficient appears in the geometrical term proportional to the Ricci scalar $R$.

## 7. Conclusions

Starting from the extended 28-component representation of the Lorentz group for a spin-1/2 particle, we have constructed a new relativistic equation that incorporates, in addition to the electric charge, three further electromagnetic characteristics. The derivation leads to a generalized 4-component Dirac-like equation in which three new interaction terms appear explicitly. Each of these additional terms can be naturally interpreted as corresponding to one of the new electromagnetic characteristics of the spin-1/2 particle, thereby extending the range of possible interactions beyond those described by the standard Dirac formalism.

The approach has been further generalized to the case of a Riemannian space–time background, where the formulation is carried out within the tetrad formalism. In this generalized setting, the presence of curvature introduces a number of additional geometrical interaction terms into the basic equation. These terms involve contributions from the Ricci scalar $R(x)$, the Ricci tensor $R_{\alpha\beta}(x)$, and the Riemann curvature tensor $R_{\alpha\beta\rho\sigma}(x)$. The resulting framework thus provides a unified description of spin-1/2 particles with anomalous electromagnetic properties, applicable in both flat and curved space–time, and explicitly shows how electromagnetic and geometrical interactions can be incorporated simultaneously into the relativistic dynamics of the particle.

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## ԱՄՓՈՓԱԳԻՐ

# Երեք լրացուցիչ բնութագրերով 1/2 սպին ունեցող մասնիկի համար նոր հավասարում էլեկտրամագնիսական և գրավիտացիոն դաշտերի դեպքում

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Գելֆանդ–Յագլոմի ընդհանուր մեթոդի շրջանակներում, ելնելով Լորենցի խմբի ընդլայնված՝ 28 բաղադրիչ ունեցող ներկայացումից, մենք կառուցում ենք նոր ռելատիվիստական $P$-ինվարիանտ ընդհանրացված հավասարում 1/2 սպին ունեցող մասնիկի համար, որը էլեկտրական լիցքից բացի, օժտված է նաև երեք բնութագրերով: Մոդելը նախ կառուցվում է ազատ մասնիկի համար, որի դեպքում ստացվում է համապատասխան սպինորային հավասարումների համակարգ, որն այնուհետև վերափոխվում է սպին–թենզորային ձևի: Ապա այս ներկայացման մեջ ներառվում է արտաքին էլեկտրամագնիսական դաշտերի հետ փոխազդեցությունը: Լրիվ ալիքային ֆունկցիայի լրացուցիչ փոփոխականների արտաքսման արդյունքում ստացվում է նվազագույն՝ չորս բաղադրիչ ունեցող Դիրակի տիպի հավասարում, որը պարունակում է փոխազդեցության երեք նոր անդամ՝ մեկնաբանվող որպես մասնիկի լրացուցիչ էլեկտրամագնիսական բնութագրերից ծագող: Այս մոտեցումը հետագայում ընդլայնվում է Ռիմանի տարածաժամանակային ֆոնի վրա՝ ավանդական տետրադային ֆորմալիզմի շրջանակներում, ինչի արդյունքում առաջանում են երկրաչափական փոխազդեցության լրացուցիչ անդամներ՝ ներառյալ Ռիչիի $R(x)$ սկալարը, Ռիչիի $R_{\alpha\beta}$ թենզորը և Ռիմանի $R_{\alpha\beta\rho\sigma}(x)$ կորության թենզորը:

**Բանալի բառեր՝** 1/2 սպին ունեցող մասնիկ, ռելատիվիստական համաչափություն, ընդհանրացված հավասարում, լրացուցիչ էլեկտրամագնիսական բնութագրեր, արտաքին էլեկտրամագնիսական և գրավիտացիոն դաշտեր